Forecasting with Real Business Cycle Models

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Theme: Forecasts with a real business cycle model

Forecasts: “statistical” models

Real business cycle models: comovements

Early attempts

Rotemberg & Woodford (1997)

One state variable

Hansen & Prescott (1993)

One specific episode

Here:

- 1960–1991
- 4 countries
  - USA, Canada, Japan, Europe
- 3 state variables
  - Solow residuals
  - Public expenditures
  - Private physical capital

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U(c_{it},1-n_{it})\right],\quad 0>\beta>1$$

$$U(c_{it},1-n_{it})=\left[c_{it}^{\mu_i}(1-n_{it})^{1-\mu_i}\right]^{\gamma}/\gamma,\quad 0<\mu_i<1,\quad \gamma<1$$

$$y_{it}=z_{it}F(k_{it},n_{it})$$

$$F(k_{it},n_{it})=k_{it}^{1-\theta_i}n_{it}^{\theta_i},\qquad 0<\theta_i<1$$

$$c_{it}+x_{it}+g_{it}=y_{it}$$

$$k_{i,t+1}=(1-\delta_i)k_{it}+x_{1it},\quad 0<\delta_i<1$$

$$z_{i,t+1}=\rho_{iz}z_{it}+\varepsilon_{zi,t+1}$$

$$\varepsilon_{zi,t+1}\rightsquigarrow\mathcal{N}(\not\prec_{\ddagger\rangle},\sigma^{\in}_{\ddagger\rangle})$$

$$g_{i,t+1}=\rho_{ig}g_{it}+\varepsilon_{gi,t+1}$$

$$\varepsilon_{gi,t+1}\rightsquigarrow\mathcal{N}(\not\prec_{\}\rangle},\sigma^{\in}_{\}\rangle})$$

$$
\begin{aligned}
z_{t+1} &= A(L)z_t + B(L)\varepsilon_{t+1} \\
\text{where} & \\
z_t &= \begin{pmatrix} z_{1t} \\ z_{2t} \\ z_{3t} \\ z_{4t} \end{pmatrix}, \quad \varepsilon_{t+1} = \begin{pmatrix} \varepsilon_{1,t+1} \\ \varepsilon_{2,t+1} \\ \varepsilon_{3,t+1} \\ \varepsilon_{4,t+1} \end{pmatrix} \\
A(L) &= \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \\
B(L) &= \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix} \\
\varepsilon_{t+1} &\rightsquigarrow \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} v_{11} & v_{21} & v_{31} & v_{41} \\ v_{21} & v_{22} & v_{32} & v_{42} \\ v_{31} & v_{32} & v_{33} & v_{43} \\ v_{41} & v_{42} & v_{43} & v_{44} \end{bmatrix} \right)
\end{aligned}
$$

$$
\begin{aligned}
g_{t+1} &= C(L)g_t + D(L)\nu_{t+1} \\
\text{where} & \\
g_t &= \begin{pmatrix} g_{1t} \\ g_{2t} \\ g_{3t} \\ g_{4t} \end{pmatrix}, \quad \nu_{t+1} = \begin{pmatrix} \nu_{1,t+1} \\ \nu_{2,t+1} \\ \nu_{3,t+1} \\ \nu_{4,t+1} \end{pmatrix} \\
C(L) &= \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{pmatrix}, \\
D(L) &= \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{pmatrix} \\
\nu_{t+1} &\rightsquigarrow \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} w_{11} & w_{21} & w_{31} & w_{41} \\ w_{21} & w_{22} & w_{32} & w_{42} \\ w_{31} & w_{32} & w_{33} & w_{43} \\ w_{41} & w_{42} & w_{43} & w_{44} \end{bmatrix} \right)
\end{aligned}
$$

Calibration

$$\max_{\{k_{i,t+1},x_{it},n_{it},j=1,4\}_{t=0}^{\infty}}$$

$$E_0\left[\sum_{t=0}^{\infty}\beta^t U(z_{it}F(k_{it},n_{it})-x_{it}-g_{it},1-n_{it})\right]$$

S.T.

and the laws of motion

$$
\begin{aligned}
n_i &= \frac{(1-\theta_i)\mu_i}{(1-\mu_i)c_i + (1-\theta_i)\mu_i}, \\
r_i &= \frac{\theta_i}{q_i}\frac{1}{k_i} - \delta_i, \\
\beta &= \frac{1}{1+r_i}, \\
\delta_i &= \frac{x_i}{k_i}, \\
z_i &= \frac{1}{k_i^{\theta_i} n_i^{1-\theta_i}},
\end{aligned}
$$

where

$$
\begin{aligned}
c_i &= 1 - x_i - g_i, \\
q_i &= \frac{1}{J}\sum_{j=1}^{J}(1+r_i)^{j-1}.
\end{aligned}
$$

Extensions:

- Compare with an open economy
- Introduce adjustment costs for capital
- Sensitivity analysis
- Other statistics