Objective of the paper

Can a DSVAR or LSVAR observe that hours increase after a positive technology shock, as a standard RBC model would indicate?

Why important?

Galí (1999), Francis and Ramey (2003), Galí and Rabanal (2004)
vs.
Christiano, Eichenbaum and Vigfusson (2003)

Major issues

Can a VAR identify correctly a model?

Are samples long enough to correctly specify the lag structure of a VAR?

Main results

- DSVAR are fundamentally misspecified for identifying a standard RBC model and lead to wrong inferences
- Standard lag length tests do not detect the need for more lags
- 2) is the reason for 1) (?)
- LSVAR are essentially uninformative
- Longer data samples would lead to wrong inferences

What is a VAR?

$$y_t = \alpha + \sum_{j=1}^{\infty} V_j y_{t-j} + a_t$$

where $a_t = y_t - E(Y_t|y^{t-1})$, $\sum_{j=1}^{\infty} \text{tr}(V_j V_j') < +\infty$, $Ea_t a_t\prime = \Omega$, $Ea_t = 0$, $Ea_t y_{t-j}' = 0$ for $j \geq 1$.

Take $\varepsilon_t | E\varepsilon_t = 0, E\varepsilon_t\varepsilon_t' = I, E\varepsilon_t\varepsilon_t' = 0 \quad \forall j \neq 0$. Then $\exists$ a unique $G|GQ\varepsilon_t = a_t$ for any orthonormal Q. Let $Q = I$.

Thus

$$y_t = \mu_y + c(L)G\varepsilon_t$$

with $c(L) = (I - \sum_{j=1}^{\infty} V_j L^j)^{-1}$.

What is the tested model?

$$x_{t+1} = Ax_t + Bw_t$$
$$y_t = Cx_t + Dw_t$$

with $Ew_t = 0$, $Ew_t w_t' = I$, $Ew_t w_{t-j} = 0$ for $j \neq 0$.

Fernández-Villaverde, Rubio-Ramírez and Sargent (2004): this can be represented as

$$\hat{x}_{t+1} = A\hat{x}_t + KG\varepsilon_t$$
$$y_t = C\hat{x}_t + G\varepsilon_t$$

with $\hat{x}_t = E(x_t|y^{t-1})$, $G\varepsilon_t \equiv a_t = y - E(y_t|y^{t-1})$, K is Kalman gain.

Thus

$$G\varepsilon_t = \begin{bmatrix} C & -C \end{bmatrix} \begin{bmatrix} x_t \\ \hat{x}_t \end{bmatrix} + Dw_t$$

and

$$\begin{bmatrix} x_{t+1} \\ \hat{x}_{t+1} \end{bmatrix} = \begin{bmatrix} A & 0 \\ KC & A - KC \end{bmatrix} \begin{bmatrix} x_t \\ \hat{x}_t \end{bmatrix} + \begin{bmatrix} B \\ KD \end{bmatrix} w_t.$$

Let

$$A^* \equiv \begin{bmatrix} A & 0 \\ KC & A - KC \end{bmatrix}$$

Then

$$G\varepsilon_t = \left(D + \begin{bmatrix} C & -C \end{bmatrix} [I - A^* L]^{-1} \begin{bmatrix} B \\ KD \end{bmatrix} L \right) w_t,$$

or

$$G\varepsilon_t = \sum_{j=0}^{\infty} h_j w_{t-j}$$

if eigenvalues of $A - KC$ are < 1 in modulus. This is verified, for #shocks=#variables, if:

- D is invertible;
- A is stable;
- the eigenvalues of $A - BD^{-1}C$ are < 1.

$$\begin{bmatrix} 1 \\ \log k_{t+1} \\ \log z_t \\ \tau_{lt} \\ \log \frac{y_t}{l_t} \\ \log l_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ cst & \gamma_k & \gamma_z & \gamma_l & 0 & 0 \\ \mu_z & 0 & 1 & 0 & 0 & 0 \\ (1-\rho)\bar{\tau}_l & 0 & 0 & \rho & 0 & 0 \\ cst & \theta(1-\phi_k) & 1-\theta(1+\phi_z) & -\theta\phi_l & 0 & 0 \\ cst & \phi_k & \phi_z & \rho\phi_l & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \log k_t \\ \log z_{t-1} \\ \tau_{lt-1} \\ \log \frac{y_{t-1}}{l_{t-1}} \\ \log l_{t-1} \end{bmatrix} +$$

$$\hookrightarrow \begin{bmatrix} 0 & 0 \\ \sigma_z & 0 \\ 0 & \sigma_l \\ 0 & 0 \\ (1-\theta(1+\phi_z))\sigma_z & -\theta\phi_l\sigma_l \\ \phi_z\sigma_z & \phi_l\sigma_l \end{bmatrix} \begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^l \end{bmatrix} \begin{matrix} \langle \\ \rangle \\ \langle \\ \rangle \\ \langle \\ \rangle \end{matrix} \begin{bmatrix} (1-\theta(1+\phi_z))\sigma_z & -\theta\phi_l\sigma_l \\ \phi_z\sigma_z & \phi_l\sigma_l \end{bmatrix} \begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^l \end{bmatrix}$$

$$\begin{bmatrix} \Delta \log \frac{y_t}{l_t} \\ \Delta \log l_t \end{bmatrix} = \begin{bmatrix} cst & \theta(1-\phi_k) & 1-\theta(1+\phi_z) & -\theta\phi_l & -1 & 0 \\ cst & \phi_k & \phi_z & \rho\phi_l & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ \log k_t \\ \log z_{t-1} \\ \tau_{lt-1} \\ \log \frac{y_{t-1}}{l_{t-1}} \\ \log l_{t-1} \end{bmatrix} + \hookleftarrow$$

$$\begin{bmatrix} 1 \\ \log k_{t+1} \\ \log z_t \\ \tau_{l\,t} \\ \log \frac{y_t}{l_t} \\ \log l_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & .968 & -.962 & -.067 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & .969 & 0 & 0 \\ 0 & .451 & .549 & .540 & 0 & 0 \\ 0 & -.287 & .287 & -1.50 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \log k_t \\ \log z_{t-1} \\ \tau_{l\,t-1} \\ \log \frac{y_{t-1}}{l_{t-1}} \\ \log l_{t-1} \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ .0089 & 0 \\ 0 & .0098 \\ .0049 & .0053 \\ .0026 & -.015 \end{bmatrix} \begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^l \end{bmatrix}$$

$$\begin{bmatrix} \Delta \log \frac{y_t}{l_t} \\ \Delta \log l_t \end{bmatrix} = \begin{bmatrix} 0 & .451 & .549 & .540 & -1 & 0 \\ 0 & -.287 & .287 & -1.50 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ \log k_t \\ \log z_{t-1} \\ \tau_{l\,t-1} \\ \log \frac{y_{t-1}}{l_{t-1}} \\ \log l_{t-1} \end{bmatrix} + \begin{bmatrix} .0049 & .0053 \\ .0026 & -.015 \end{bmatrix} \begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^l \end{bmatrix}$$

DSVAR characteristics:

- A stable
- D invertible
- eigenvalues of $A - BD^{-1}C$: $\begin{bmatrix} -.4064 \\ .0084 \\ 1 \\ 1.3614 \end{bmatrix}$

$$
\begin{bmatrix} 1 \\ \log k_{t+1} \\ \log z_t \\ \tau_{l\,t} \\ \log \frac{y_t}{l_t} \end{bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
cst & \gamma_k & \gamma_z & \gamma_l & 0 \\
\mu_z & 0 & 1 & 0 & 0 \\
(1-\rho)\bar{\tau}_l & 0 & 0 & \rho & 0 \\
cst & \theta(1-\phi_k) & 1-\theta(1+\phi_z) & -\theta\phi_l & 0 \\
cst & \phi_k & \phi_z & \rho\phi_l & 0
\end{bmatrix}
\begin{bmatrix} 1 \\ \log k_t \\ \log z_{t-1} \\ \tau_{l\,t-1} \\ \log \frac{y_{t-1}}{l_{t-1}} \end{bmatrix}
+
$$

$$
\hookrightarrow
\begin{bmatrix}
0 & 0 \\
\sigma_z & 0 \\
0 & \sigma_l \\
0 & 0 \\
(1-\theta(1+\phi_z))\sigma_z & -\theta\phi_l\sigma_l \\
\phi_z\sigma_z &
\end{bmatrix}
\begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^l \end{bmatrix}
\begin{matrix} \langle \\ \rangle \\ \langle \\ \rangle \\ \langle \end{matrix}
\begin{bmatrix}
(1-\theta(1+\phi_z))\sigma_z & -\theta\phi_l\sigma_l \\
\phi_z\sigma_z & \phi_l\sigma_l
\end{bmatrix}
\begin{bmatrix} \varepsilon_t^z \\ \varepsilon_t^l \end{bmatrix}
$$

$$
\begin{bmatrix} \Delta \log \frac{y_t}{l_t} \\ \log l_t \end{bmatrix}
=
\begin{bmatrix}
cst & \theta(1-\phi_k) & 1-\theta(1+\phi_z) & -\theta\phi_l & -1 \\
cst & \phi_k & \phi_z & \rho\phi_l & 0
\end{bmatrix}
\begin{bmatrix} 1 \\ \log k_t \\ \log z_{t-1} \\ \tau_{l\,t-1} \\ \log \frac{y_{t-1}}{l_{t-1}} \end{bmatrix}
+ \hookleftarrow
$$

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$$\begin{bmatrix} \Delta \log \frac{y_t}{l_t} \\ \log l_t \end{bmatrix} = \begin{bmatrix} 0 & .451 & .549 & .540 & -1 \\ 0 & -.287 & .287 & -1.50 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \log k_t \\ \log z_{t-1} \\ \tau_{lt-1} \\ \log \frac{y_{t-1}}{l_{t-1}} \end{bmatrix} +$$

LSVAR characteristics:

- A stable
- D invertible
- eigenvalues of $A - BD^{-1}C$: $\begin{bmatrix} -.4064 \\ .0084 \\ 1 \\ 1.3614 \end{bmatrix}$

Small sample properties:

- Illusion of degrees of freedom
- Unit root tests have low power
- Root close to one: bad small sample properties of impulse responses (Pesavento and Rossi (2003))